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Citizen Math: Using Math Class to Create Informed, Thoughtful, and Productive Citizens

How can we better prepare students to engage in constructive civil discourse given the explosion of information, rapid advances in technology, and our ever-changing society? Understanding complex public policies, such as the consequences of a higher minimum wage, require students to apply mathematical knowledge, reason quantitatively, construct viable arguments, and consider different perspectives with civility. Citizen Math lessons provide real, relevant questions of societal and personal importance that enable students to develop critical civics and social-emotional learning skills (such as critical thinking, complex problem solving, and collaboration) by applying rigorous grade level mathematics. WestEd and AIR are enthusiastic to partner with Citizen Math to scale and establish the efficacy of a promising innovation to improve outcomes for high-need students in rural and urban settings across three states. By bringing Citizen Math to scale, students across the country will have the opportunity to develop the skills and dispositions necessary to becoming informed, thoughtful, and productive citizens.

The proposed project addresses both **Absolute Priority 1 (Moderate Evidence)** and **Absolute Priority 4 (Fostering Knowledge and Promoting the Development of Skills that Prepare Students to be Informed, Thoughtful, and Productive Individuals and Citizens)**. To meet Absolute Priority 1, Citizen Math is well-aligned with four of the five recommendations in the What Works Clearinghouse Practice Guide, *Improving Mathematical Problem Solving in Grades 4 Through 8* (Woodward et al., 2018). The program provides students with ongoing opportunities to monitor and reflect on their problem-solving process (strong evidence), use visual representations (strong evidence), see and apply multiple problem-solving strategies (moderate evidence), and recognize and articulate mathematical concepts and

notation (moderate evidence). In addition, a multisite randomized controlled trial found that middle school students whose teachers used Citizen Math lessons significantly outperformed a control group on the state math test. Citizen Math students also scored significantly higher than the control group on perceptions of math as an interesting subject with real world applications and of their math teachers as promoting deep understanding of the content (Jackson and Markarin, 2018). To meet Absolute Priority 4, Citizen Math lessons prepare students for a world and workplaces that value teams who can work well and persistently together, deal effectively with conflict and uncertainty, craft evidence-based arguments respectfully, and propose solutions to complex societal problems that require nuanced solutions and understanding multiple points of view.

The three objectives of this project are to: (1) create and test strategies to enhance the adoption, use, and sustainability of Citizen Math; (2) implement and evaluate the efficacy of Citizen Math on students' academic outcomes, attitudes, and perceptions in rural and urban districts predominantly serving high-need students; and (3) disseminate findings and track the scaling of this promising program.

A. SIGNIFICANCE

A.1. National Significance

A healthy American democracy depends on an informed, thoughtful, and engaged citizenry. Yet many civic indicators are troubling. Public trust in the federal government and opportunities for civic engagement are both in rapid decline (Pew Research Center, 2019; Atwell, Bridgeland, and Levine, 2017) and political polarization is on the rise (Pierson & Schickler, 2020; Rekker, 2021). Such polarization and animosity are evident on the radio, cable television, and across the internet, where online communities often further exacerbate these

divisions (e.g., Rusciano & Louis, 2014; Barberá, 2020; Yarchi, Baden & Kligler-Vilenchik, 2021). Addressing the root causes of these disturbing trends has no easy answers, but education is likely to be part of any successful reform effort. As President John F. Kennedy remarked in response to the familiar saying, the course of civilization is a race between catastrophe and education, "we must make sure that education wins." (NAEP, 2018). Schools and classrooms enable students from different backgrounds and perspectives to learn not only academic content, but also how to collaborate with each other, listen to other points of view, and sharpen their own thinking—all fundamental to civil discourse.

Though it is sobering that a majority of adults cannot name the branches of government (e.g., Annenberg Public Policy Center, 2019), *civic knowledge* is only one of three essential components of the Civics Framework for the National Assessment of Educational Progress (NAEP), developed by a nonpartisan Congressional governing board. The second component is *intellectual and participatory skills*, which include "identifying and describing; explaining and analyzing; and evaluating, taking, and defending positions." The third component is *civic dispositions*, which include "respecting individual worth and human dignity and participating in civic affairs in an informed, thoughtful, and effective manner." (NAEP, 2020). Results from recent assessments indicate that the latter two components warrant reform. On the most recent NAEP civics assessments, 75% of eighth graders scored failed to reach proficiency, and performance was worse on questions that measured participatory skills (e.g., explaining citizen involvement or the difference that volunteers can make) compared to civic knowledge (e.g., explaining how a congressional veto works). (NAEP, 2019). On the most recent Program for International Student Assessment (PISA), U.S. 15-year-olds performed well below the international average in math problem solving skills and only 9% were able to accurately

distinguish a fact from an opinion (Organisation for Economic Co-operation and Development, 2019).

Though these data are discouraging, there is reason for some optimism. Meta-analyses of programs designed to enhance personal and social skills found improvements in both social-emotional skills and academic achievement (Durlack, et al., 2011) and the effects were lasting (Taylor, et al., 2017). Further, "deeper learning" in which students know how, why, and when to apply knowledge to answer questions and solve problems is much more likely with instructional activities designed to teach inter and intrapersonal competencies, foundational to the productive citizenship (NRC, 2012). Presenting and discussing content that is interesting and relevant to students and their local communities helps students learn content and how to debate in an informed, respectful way (ECS, 2014).

A.2. Citizen Math: A Uniquely Inspiring, Evidence-Based, Practical Solution

Citizen Math integrates social, emotional, and academic learning skills that prepare students to be informed, thoughtful, and productive individuals and reflects features of teaching and learning advocated by both math education and civic education reformers. In each Citizen Math lesson, students develop problem solving and communication skills by *applying* math to explore and discuss engaging questions relevant to students' lives. Across lessons, students analyze pressing and complex societal issues (e.g. *How are rising incomes related to homelessness and how should cities respond?*); reflect on their inner worlds (e.g. *Is there an upside to negative emotions...and what does an ideal emotional week look like for me?*); and reimagine conventions (e.g. *Should people with large feet pay more for shoes?*). Students work collaboratively and respectfully as they use math to understand and discuss issues that lack a single correct response.

A recent article in Education Week titled, “Math: The Most Powerful Civics Lesson You've Never Had,” highlights how a Citizen Math teacher used math to help students deepen their understanding of the world (Sawchuk, 2019). She used a Citizen Math lesson on solar power to frame a cost-benefit analysis about the solar panels recently installed at their school which led to a discussion about whether the school’s planned energy production was likely to meet the renewable energy guidelines for 2030 established by the United Nations. The teacher said, “We live in a bubble on social media with people with like-minded opinions. I want them to open it up and see both sides to arguments, and be able to support theirs with more than an opinion.” She added, “And in my classroom, the support is the math.”

Citizen Math activities are well aligned with WWC evidenced-based problem-solving practices (Woodward et al., 2018) and produced positive effects on student mathematics achievement in a single year, even when teachers used as few as two lessons (Jackson and Makarin, 2018). In contrast, studies of promising full year middle school math curricula failed to produce discernible effects, e.g., a study of the Connected Mathematics Project (U.S. Department of Education, What Works Clearinghouse, 2017).

Citizen Math is also affordable and practical. District subscriptions to Citizen Math amount to only a few dollars per student per year, in contrast with some textbooks with digital supplements that can run upwards of $80 per student per year. Citizen Math has numerous lessons[1] aligned with state standards for Grades 6, 7, 8, Algebra 1, and Algebra 2 that easily integrate with any curriculum, pacing guide, and instructional unit.

[1] Citizen Math uses definitions for content areas that are common across most states’ content standards. Lessons are organized by topics using names generated with teachers to be universally recognizable.

Citizen Math has rigorous evidence of effectiveness, low cost, and is easily implemented across middle and high school. The proposed project will determine whether a novel, integrated, and practical instructional approach can successfully improve outcomes in both math and social-emotional skills that form the foundation of strong citizenship. The project will support successful scaling by addressing barriers to scale, such as cost and implementation fidelity and will build evidence of effectiveness, meeting strong level of evidence as defined in the notice.

B. STRATEGIES TO ADDRESS BARRIERS TO SCALE

B.1. Strategies to Address Barriers to Scale

More than 4,000 teachers used at least one Citizen Math lesson in the past year, reaching more than 200,000 students. However, the majority of these teachers are highly motivated, early adopters with access to the lessons outside of a district license. These teachers either pay for a license out of pocket, receive individual access from their school site, or use the few, free, sample lessons. We have identified three barriers to scale: 1) teachers with more traditional approaches to math instruction may need more professional learning than the lesson guides currently provide; 2) teachers working in rural or underserved communities may lack peers who can share best practices and suggestions for implementation given the pragmatic constraints of their settings; and 3) districts have particular needs (often visible in their RFPs) for new curricular supports, such as integration with learning management systems, alignment with approved scope and sequences, and translation into multiple languages. To address these barriers, we propose three strategies: to create 1) flexible, online professional learning opportunities, 2) an online teacher community, and 3) additional supports for district adoption.

Strategy 1. Create Additional Flexible, Online Professional Learning Opportunities

Though many teachers use Citizen Math without any formal training, districts that provide structured professional learning have more teachers use the lessons more frequently and develop a culture of practice around the lessons. However, face-to-face professional learning is costly, time-consuming, requires travel, and Citizen Math currently receives more requests for in-person workshops than they can feasibly lead. To remedy these challenges, we propose to develop virtual workshops, video case studies, and on-demand, lesson-specific videos.

Virtual professional learning workshops. During the early years of the project, we will iteratively develop and test online synchronous and asynchronous models for professional learning. During the virtual workshops, teachers will (a) learn the philosophy behind social and emotional learning-infused mathematics and authentic applications that prepare students to be informed, thoughtful, and productive individuals; (b) participate in or observe a Citizen Math lesson complete with rich mathematical discourse, collaboration, perspective-taking, and debate; and (c) engage in reflection, practice, and planning. We will pilot different versions of the workshops with groups of teachers, gather feedback on satisfaction and the utility of these sessions, and apply learnings to the professional learning offered during the impact study.

Video cases and lesson-specific supports. A critical component of professional learning is modeling best practices. When teachers witness well-taught Citizen Math lessons, they see how the activities elicit students' excitement, passion, and curiosity; experience the complex problem-solving and collaboration; and hear the types of prompts that lead to deeper analysis and productive debate. Working with experienced teachers, we propose to create five case studies that feature classroom footage full of lively student discourse to provide teachers with a vision of Citizen Math at each grade level. We will additionally create five to ten short

videos that showcase rich moments from various lessons to demonstrate particular instructional strategies implemented with high quality that apply across lessons (e.g., how to productively address math misconceptions while keeping the conversation rooted in the real-world issue).

Strategy 2. Establishing and Fostering a Citizen Math Community

When schools incorporate Citizen Math into their professional learning structures, they report significant shifts in their instructional cultures; school becomes a place where students explore and debate authentic questions about the world to become more engaged citizens. Though many teachers are local advocates for these lessons and connect with each other over social media, the lack of formal structure for *collaboration* is particularly problematic for rural teachers without access to local professional learning communities. We propose to develop a community platform that will encourage informal and formal collaborations led by Citizen Math and a cultivated group of experienced Citizen Math teachers. The platform will host the professional learning resources developed in Strategy 1 and will encourage teachers to learn from each other, provide a place for Citizen Math staff to offer technical support, and foster a sense of community with teachers across the country. Citizen Math will work with the formative evaluation team at WestEd to survey and interview groups of existing users to identify candidate platforms, invite existing teachers to test the features of the platform, and use improvement science methods to test ways of promoting engagement between users. During the impact study, study teachers will be encouraged to participate in the virtual community.

Strategy 3. Additional Supports for District Adoption

Many districts have pragmatic constraints for the curricular supports they formally adopt. Our final strategy is to carry out a needs analysis to identify common requirements and create additional supports. Currently, many districts require student-facing materials to be

translated into other languages, aligned with adopted scope and sequence guidelines, or readily implemented into existing learning management systems. Based on the data we collect, we will translate student handouts into the most commonly requested languages, create alignments with widely adopted curricula (such as open educational resources and those used by the study partners), and facilitate integration of the activities with the most used learning management systems (e.g., Google Classroom, Clever).

B.2. Mechanisms for Broad Dissemination, Further Development, or Replication

Citizen Math will lead a comprehensive campaign to communicate information to practitioners and the general public about the potential for SEL-infused math materials to support student learning. The dissemination will include articles, videos, and blog posts distributed through email, social media, journals, and news outlets. We propose to create and share *Stories from the field* to promote the idea that math is a tool for exploring the world and that math class is a place for conversations that promote citizenship. With permission, and beginning in the second year of the project, we will showcase classrooms that have used Citizen Math lessons to engage in rich debate and exploration and even extended lessons into inspiring community projects.

To reach research, policy, and practitioner audiences interested in issues related to implementation and impact, we additionally plan to present research findings at national research conferences (e.g., American Educational Research Association, Society for Research on Educational Effectiveness, National Social and Emotional Learning Conference, etc.) and publish different aspects of the findings in peer-reviewed journals (e.g., Journal of Research on Educational Effectiveness, Educational Researcher, etc.). In addition, we will submit presentations with at least one math leader from Chicago Public Schools, West Virginia, and

Maine to national practitioner or policy annual conferences (e.g., National Council of Teachers of Mathematics, Innovative Schools Summit, National Rural Education Association, etc.).

C. QUALITY OF PROJECT DESIGN

C.1. Project Objectives, Outcomes, Measures

The project aims to scale and evaluate the efficacy of Citizen Math in rural and urban schools serving high-need students. We expect the project will reach more than 50,000 middle school students across 70 rural schools in Maine and West Virginia, and 50 urban schools in Chicago. To achieve our aims, we propose to (1) create and test strategies to enhance adoption, use, and sustainability; (2) implement and test the efficacy of Citizen Math in our high-need sample; and 3) disseminate findings and track progress on scaling. Exhibit 1 details the strategies, outcomes, and measures for each of our objectives.

Exhibit 1. Strategies, Outcomes, and Measures, by Project Objective

Strategies	Outcomes	Measures
Objective 1. Create and test strategies to enhance adoption, use, and sustainability of Citizen Math		
1.1. Create and test flexible teacher supports and training opportunities.	Online offerings; lesson-specific supports, including video cases.	4 virtual workshops, 5 case studies, and 5-10 short videos
1.2. Create and test a community platform and community building practices	Online Community (Platform) for teachers to collaborate, learn, and share best practices.	At least 75% of study teachers from West Virginia, Maine, and Chicago Public Schools will use the platform
1.3. Create additional supports for district adoption	Translated student materials, alignment with widely-used curricula, integration with LMS	Translation of student materials to 2 most commonly requested languages; integration with 2 commonly used LMS; and alignment with 3 commonly used curricula
Objective 2. Evaluate the implementation and impact of Citizen Math		
2.1. Create and refine materials and train recruiting team	Recruitment materials and resources; trained project recruitment staff.	IRB-approved materials; 100% of project recruitment staff report clarity on recruitment approach.
2.2. Recruit and randomly assign to treatment and control condition schools serving high-need students from partner states and district.	Principals and teachers at targeted schools agree to random assignment design. Treatment and control school	Signed memoranda of understanding from 70 schools in West Virginia and Maine and 50 schools in Chicago Public Schools.

	samples achieve baseline equivalence on primary characteristics.	
2.3. Prepare data collection instruments and procedures and train data collection staff.	Finalized data collection instruments and procedures; trained project data collection staff.	Internal project QA reviewers approve final data collection instruments; 100% of project data collection staff report clarity on data collection procedures.
2.4. Measure and assess implementation fidelity of Citizen Math.	Fidelity of implementation data collected from teacher PD meetings, Citizen Math usage, and teacher logs.	Data collection progress and summary memos indicate at least 80% of teachers are participating in PL and using at least six Citizen Math lessons.
2.5. Measure and assess the impact of Citizen Math on instruction, student achievement, and student social-emotional learning.	Teacher log data collected (from treatment and control schools) and analyzed; student outcome data collected (from treatment and control schools) and analyzed.	Data collection progress and summary memos indicate at least 75% student and at least 90% teacher response rates on all measures described in Section E. Impact memo indicates study meets WWC w/o reservation.
Objective 3. Disseminate findings and track progress on scaling		
3.1. Disseminate findings to research audiences	Project team submits a research paper proposal summarizing study results to a national research conference.	Presentation of research findings given at 2 national research conferences and published in 2 peer-reviewed journals.
3.2. Disseminate findings to public, teacher, and policy audiences	Partner math leaders and project team submit at least one proposal to a national practitioner or policy conference.	Presentations with at least one math leader from Chicago Public Schools, West Virginia, and Maine given at least one national practitioner or policy conference.
3.3. Continue to support adoption of Citizen Math and track scaling.	Offer Citizen Math to teachers, schools, and districts around the country	Monthly documentation of new, non-study schools and districts using Citizen Math in Years 3-5 of the project, with total targets of 480 schools and 20 districts by end of Year 5.

C.2. Conceptual Framework

We hypothesize that Citizen Math will develop rich social-emotional that prepare students to be informed, thoughtful, and productive individuals as well as academic skills, so students are more likely to succeed in school and life. Our logic model (see Appendix G.1), outlines how the key components and associated activities of Citizen Math lead to the expected intermediate and direct outcomes.

Key Components and Associated Instructional Activities. The key components and associated instructional activities of Citizen Math include the lessons themselves, and implementation supports and resources for teachers. Citizen Math lessons use math to explore **authentic, engaging, relevant topics** such as: *How does the media we consume affect our well-being?* (Ratios) *What's a fair way to tip?* (Percents) *How dangerous is texting and driving?* (Proportions). Lessons are well-researched and, where applicable, include real data sets for meaningful analysis. For example, in the middle school lesson Temp Work, students use integer operations and public data from temperature stations around the globe to analyze how global temperatures have changed in the past century. Citizen Math lesson designers consulted with climate scientists to ensure the appropriate metrics were used and described meaningfully.

The lessons are designed around **carefully crafted questions** that encourage engagement, reflection on problem solving (Recommendation 2 in Woodward et al., 2018) alternate perspective-taking and approaches (Recommendation 4 in Woodward et al., 2018), discussion, and deep understanding of mathematical content. Each lesson begins with a launch: a short video and a few provocative questions designed to activate prior knowledge, spark curiosity, and/or solicit opinions. As launch questions intentionally require no math knowledge, all students can participate in the discussion regardless of their historical performance or confidence in math, e.g., *What do you know about viruses and how health workers protect themselves while caring for the sick*?

After the launch, subsequent questions lead students to meaningful discovery and prepare them for informed debate. For example, in a lesson about cities' struggles with homelessness amidst rising rents, students analyze a graph of annual rents vs. median income in New York between 2010–2020, mathematically interpret the slope of the line, and discuss how

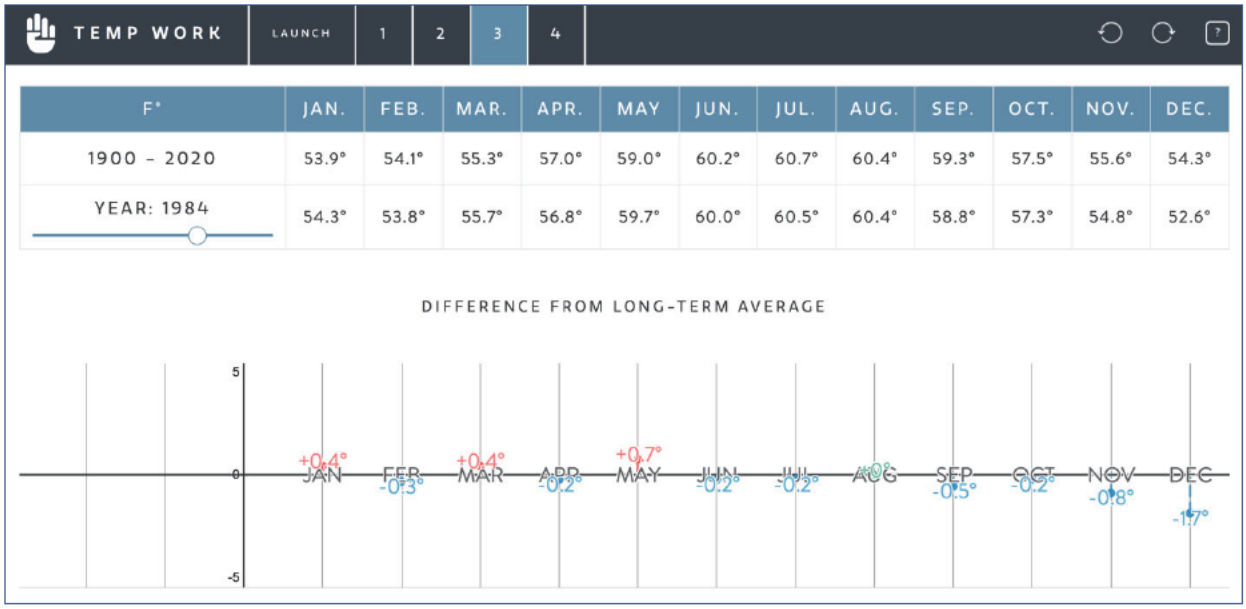
the slope affects different people living in the city. Throughout the lesson, students discover that as incomes rise, rents rise, and as rents rise, so does homelessness. The final question of the lesson challenges students to imagine they are city leaders debating whether to recruit a large tech company to move its headquarters to the area. Students are asked what factors they would consider in their decision and how they might mitigate unintended consequences. Questions encourage students to approach the problem in ways that make the most sense to them, honoring different levels of sophistication and complexity. Research suggests this approach to questioning improves student outcomes. Questions that encourage elaboration and self-explanation help students learn more deeply and support transfer (e.g., NRC, 2012, Pashler et al., 2007) and exposure to varied approaches has been shown to improve math learning (e.g., Recommendation 4 in Woodward et al., 2018; Star & Rittle-Johnson, 2008; Star & Seifert, 2006).

Citizen Math lessons emphasize **discussion-based collaborative learning.** Each lesson involves cycles of independent work, small group collaboration on hard problems, and whole class discussions. Students share their reasoning, critique the reasoning of others, and prepare presentations of their positions. Through discussions, students learn that academic culture is inherently collaborative; challenging math, learning about the world, and talking about issues that matter and debating them civilly are things we do together. Participating in collaborative discussions about complex issues gives students extensive practice in critical thinking and perspective-taking. In addition to the social benefits, engaging in rigorous, respectful discussion builds mathematical understanding (e.g., Atwood, Turnbull & Carpentale, 2010).

Each Citizen Math lesson **integrates multiple, dynamic visual representations**, such as graphs, tables, and symbolic notations, supporting students' development of conceptual

understanding (see Appendix J.1 for examples). Each lesson includes online content for the teacher to use as an instructional aid in the classroom, such as an interactive tool that highlights how the monthly global average temperature varies from the 100-year average, illustrating how months tend to be warmer than average over time (Exhibit 2). Generating and discussing visual representations of abstract mathematical ideas promotes student learning (Recommendation 3 in Woodward et al., 2018; Fennema & Franke, 1992; Goldin & Shteingold, 2001; Presmeg, 2006) and enables students to flexibly apply their knowledge (Lesh, Post, & Behr, 1987).

Exhibit 2. Interactive Representations Used in the Citizen Math Lesson, Temp Work



Lessons **focus on core grade-level mathematics concepts** aligned with state standards and the National Council of Teachers of Mathematics Principles and Standards for School Mathematics (e.g., National Council of Teachers of Mathematics, 2000; 2006). As students solve challenging problems, they develop procedural fluency in conjunction with conceptual understanding (National Research Council, 2001; Hiebert and Grouws, 2007), equipping them to describe and make sense mathematical concepts and make sense of algebraic notations (Recommendation 5 in Woodward et al., 2012). As one aim of Citizen Math lessons is for

students to use math to understand the world, teachers in the Jackson study said that they referred back to the Citizen Math lessons throughout their units because the contexts helped students make sense of the mathematical concepts.

Citizen Math has three key features to support teachers in their implementation. First, the lessons are **easy and flexible to implement** because they **align with instructional units** of any middle school curriculum. Citizen Math offers at least two lessons for each grade-level topic. By implementing a lesson early in the unit, teachers can revisit the context as student understanding is solidifying. Teachers may also use a second lesson at the end of the instructional unit, as a summative capstone activity (see Appendix J.3 for a list of two Citizen Math lessons for each of the major topics in Grades 6–8).

Clear and concise **lesson guides**, iteratively improved with user feedback, help teachers efficiently plan and implement each lesson. Guides provide ***suggested timings*** so teachers devote appropriately large chunks of time (30–45 minutes) on complex problem-solving and less time (5 minutes) on the introductory activities. ***Exemplary question responses*** provide details on possible responses from students to help teachers anticipate and seek out different approaches for students to compare, discuss, and debate (e.g., Smith & Stein, 2011). Lesson guides also provide ***key insights and guiding questions*** that teachers can use to elicit student thinking without resorting to telling them answers (Appendix J.2 has lesson guide excerpts).

The final teacher support resources will be developed through the scaling phase: **an online community of practice with on-demand resources for professional learning and collaboration** for teachers and leaders**.** Though many Citizen Math teachers start using the program with only the lesson guides, Citizen Math currently offers additional professional learning experiences and customized resources to support quality implementation, including

daylong or multi-day in person workshops to schools and districts. Through the mid-phase EIR, we propose to create an online community of practice that would allow teachers to collaborate synchronously and asynchronously during the school year, sharing challenges, solutions, and tips for specific lessons and lesson sequences in different types of instructional contexts. The platform will also host to-be-developed online resources that include video cases of exemplary lesson excerpts that highlight particular routines and rich discussions.

Connecting Key Components with Outcomes. These lesson components and instructional activities are hypothesized to improve immediate and direct outcomes for students. When students have increased opportunities to engage in meaningful and important mathematics; justify mathematics reasoning to demonstrate critical thinking and perspective-taking; engage in complex problem solving; and solve challenging problems we expect they will improve both social-emotional and academic skills. Social-emotional skills include improved motivation, engagement, and confidence in solving problems; and improved personal relationships by empathizing with other students' perspectives. Direct academic outcomes include improved problem solving skills, conceptual understanding, and mathematical proficiency of grade-level content.

C.3. Reaching Rural and Underserved Students

Citizen Math aims to increase student achievement by engaging students in using math to explore issues that are relevant to their lives. Citizen Math lessons explore issues affecting rural and underserved students, such as how the homeless population around the country is changing, whether coupons are always a good deal, and the best strategy for harvesting trees—particularly relevant to Maine and West Virginia, where the forest industry is a significant part of each respective state's economy. The program supports Chicago Public Schools, West

Virginia, and Maine math education leaders' ongoing efforts to regularly engage students in critical thinking, problem solving, and mathematical communication. Chicago Public Schools is introducing a new curriculum in the 2021-22 school year that promotes these processes, and Citizen Math lessons will reinforce this vision of instruction as instructors become familiar with the core curriculum. Through private and federal grant funding over the past three years, our university and district partners in West Virginia have created networks of secondary math teachers and instructional leaders across the state who are working to deepen student engagement in math learning. In Maine, Citizen Math aligns with the state's mathematical content and process standards and reinforces its middle grade social studies standards, which emphasize students' ability to analyze and problem solve, not just list basic facts or describe how governmental processes work.

Citizen Math is well-suited for under-resourced communities because it does not require expensive technology. Teachers need a single computer, an internet connection, and a projection system to deliver the lessons. Finally, our proposed online community of practice will give teachers opportunities to connect with peers serving similar populations of students, regardless of geographic proximity or whether they are the only math teacher in their building.

D. ADEQUACY OF RESOURCES AND QUALITY OF THE MANAGEMENT PLAN

D.1. Capacity to Bring the Project to Scale

Citizen Math, WestEd, and AIR are very capable of bringing the proposed project to scale. If funded, Citizen Math will lead the execution of the scaling strategies described in Section C. These staff members will direct community engagement, virtual professional development, video production, and communication activities, working closely with Citizen Math's project manager and Chief Operating Officer in all five years of the project. We believe

these scaling strategies will greatly increase the demand for site and district-wide license purchases and enable Citizen Math to become self-sustainable at a larger scale within five years. WestEd and AIR staff include senior personnel with expertise in mathematics education, formative evaluation, and advanced quantitative methods. These staff also have a proven track record of successfully executing projects of similar complexity, scope, and focus, including large scale, multi-site randomized controlled trials—including multiple successful trials in two of the three project sites, Chicago Public Schools and Maine.

Exhibit 3 provides details on the capabilities of the proposed staff and their project roles. Appendix J.4 provides an organizational chart, which illustrates the responsibilities of each staff member and organization. WestEd will be responsible for managing the project and progress towards the meeting objectives, leading the formative evaluation of strategies to enhance adoption and use of Citizen Math, and leading implementation fidelity analyses. AIR will be responsible for leading the independent impact evaluation. WestEd and AIR will share responsibility for recruitment, implementation monitoring, and dissemination activities for the impact study. Citizen Math will be responsible for developing strategies to enhance adoption and providing virtual training to Citizen Math treatment teachers in the impact study.

Exhibit 3. Roles, Responsibilities, and Relevant Experience of Key Project Staff

Staff and Role	Primary Responsibilities and Relevant Experience
WestEd (Lead Grantee) Key Personnel and Other Senior Staff	
Dr. [redacted] Project Director and Recruitment Task Lead (35% - Years 1-5)	Dr [redacted] is Director of Mathematics at WestEd. He has served as PI on a number of large-scale, federally funded projects designed to improve math teaching and learning, including experimental studies and partnerships with practitioners and researchers. As project director, [redacted] will serve as the key point of contact to OESE, manage the project and partner relationships, lead recruitment for the efficacy study, and contribute to implementation analyses and the dissemination of results.
Dr. [redacted] Formative Evaluation Director (29% - Years 1-5)	Dr. [redacted] is Director of Learning and Technology at WestEd. She Directed the National Center on Cognition and Mathematics Instruction, serves as PI on multiple federally-funded projects, and serves as the formative evaluator on an ongoing EIR project. As formative evaluation Director, she will monitor progress toward objectives and lead continuous improvement cycles for the strategies to address barriers to scale.

Dr. [redacted] Implementation Fidelity Lead (45% - Years 1-5)	Dr. [redacted] is a Senior Research Associate at WestEd who leads Data processing and tracks implementation fidelity across federally-funded math education projects. She will lead the development of measures, data collection processes and analyses of implementation data for the impact study.
Other Senior Recruitment and Implementation Staff	WestEd brings several experienced staff who have successfully recruited schools for large-scale studies and supported implementation thereafter. These five staff will devote between 30 and 50% of their time to the project during Years 2-4, working closely with partner schools in Maine and West Virginia: Dr. [redacted], [redacted], [redacted], [redacted], and [redacted].
AIR (Independent Evaluator) Key Personnel and Other Senior Staff	
Dr. [redacted] Impact Evaluation Director (~15% - Years 1-5)	Dr [redacted] is a Principal Researcher with 20 years of education research and evaluation experience, specializing in research design and quantitative methodology. He will oversee all aspects of the impact evaluation.
[redacted] Impact Data Collection and Analysis Lead (~26% - Years 1-5)	[redacted] is a Researcher with 9 years of experience leading and conducting data collection and analyses in education research and evaluation projects with a focus on experimental and quasi-experimental designs. He will serve as data collection and analysis lead, as well as support Dr. [redacted] with project management.
Other Senior Evaluation, Recruitment, and Implementation Staff	Dr. [redacted] is a Principal Researcher and methodologist who will serve as senior adviser and quality assurance reviewer. Dr. [redacted] is a Principal Researcher with extensive experience recruiting schools in Chicago Public Schools who will help lead recruitment in this district. [redacted] is a Senior Researcher and social and emotional learning (SEL) expert who will support instrument development of SEL constructs.
Citizen Math Key Personnel	
[redacted] Intervention Lead (68% Years 1-2; 45% Year 3; 23% Years 4-5)	[redacted] has been building Citizen Math (formerly Mathalicious) alongside the founder since 2011 and has a track record of leading small teams to ambitious outcomes with limited resources. She will hire and lead the small new Community & Professional Learning Team within Citizen Math to design, develop, and manage the new resources proposed for scaling including: virtual workshops, on demand professional learning videos, and the online community. She will manage CM's partnership with WestEd and overall commitment to the EIR grant.

D.2 Management Plan to Meet Project Objectives

WestEd will lead the proposed project, collaborating closely with AIR, Citizen Math, and our educators in our three partner sites to meet project objectives. Exhibit 5 provides a high-level project timeline; Exhibit 6 lists the project objectives by year and responsible organization(s). Bolded organization indicates lead. Both WestEd and AIR are bolded in 2.5 because WestEd will lead implementation fidelity analyses and AIR will lead service contrast

analyses as part of independent evaluation. Appendix J.5 provides a highly detailed timeline, with objectives and responsibilities designated for each month of the five-year project.

Exhibit 4. High-Level Project Timeline, by Objective

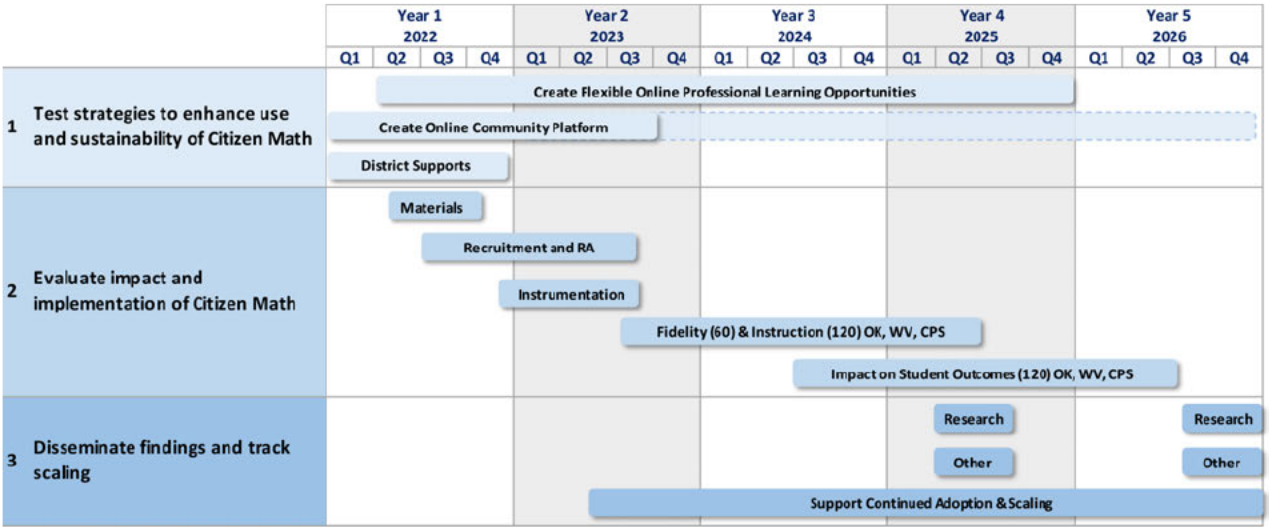


Exhibit 5. Project Objectives and Responsibilities, by Organization and Project Year

Milestones	Y1	Y2	Y3	Y4	Y5	Responsible Organization?
Objective 1: Create and test strategies to enhance adoption, use, and sustainability of Citizen Math						
1.1. Create flexible, online professional learning opportunities	x	x	x	x		**CM**, WE
1.2. Create and test a community platform that is easily accessible	x	x				**WE**, CM
1.3. Create additional supports for district adoption	x					**CM**, WE
Objective 2: Evaluate implementation and impact of Citizen Math						
2.1. Create and refine materials and train recruiters	x	x				**WE**, AIR, CM
2.2. Recruit and randomly assign schools to condition	x	x				WE, **AIR**
2.3. Prepare data collection instruments and staff		x				WE**, AIR**
2.4. Assess implementation fidelity/instr. practices			x	x	x	**WE**, **AIR**
2.5. Assess impact of Citizen Math			x	x	x	**AIR**
Objective 3: Disseminate findings and track progress on scaling						
3.1. Disseminate findings to research audiences				x	x	**AIR**, WE
3.2. Disseminate findings to other audiences				x	x	**WE**, AIR, CM

3.3. Support continued adoption and track scaling		x	x	x	x	**WE**, CM

D.3. Reasonableness of Costs in Relation to Objectives, Design, and Significance

The costs for this project are reasonable given the stated objectives and potential significance, especially considering that the products of this project will continue to support student learning far beyond the life of the grant. For example, the classroom video case studies created for this project will anchor teacher professional learning for years to come. WestEd has video-based professional learning programs that incorporate high-quality videos from lessons more than two decades old. The online professional learning community has initial startup costs, but the project team will learn how best to encourage active participation and engagement, and the platform will be available to practitioners into the future. Because the project involves educators from diverse settings, learnings about maximum uptake will be applicable to teachers in both rural and urban environments. The emphasis on creating online supports that can be easily integrated into existing learning management systems also enhances future adoption and use. The rigorous efficacy study of a concrete, supplemental program that supports social-emotional learning in a core academic subject will contribute to the evidence base in the fields of social-emotional learning, math education. Further, the evidence has the potential to inform other disciplines, such as civics and other social studies classes, as well as school-based initiatives that encourage community engagement. The size and scope of the efficacy study means that results will be generalizable for both rural and urban schools serving high-need students, rather than a single population, as well as for one and two years of program implementation.

E. QUALITY OF THE PROJECT EVALUATION

The AIR Evaluation Team will conduct an independent evaluation of the impact and implementation of Citizen Math in middle grade (6–8) math classes (see Exhibit 6). Research questions (RQ) 1–6 study the impact of the intervention on teacher and student outcomes after one year and two years of implementation. Questions 7 and 8 address implementation and are designed to provide formative feedback to guide the replication and scale-up of Citizen Math.

Exhibit 6. Evaluation Research Questions, Data Sources, and Sample

Research Question	Primary Data Source(s)
Impact analyses	
1. What is the impact of Citizen Math on the nature of teachers' instructional activities?	Study-administered teacher log
2. What is the impact of Citizen Math on students' opportunities to learn?	Study-administered student survey
3. What is the impact of Citizen Math on students' social-emotional learning?	
4. What is the impact of Citizen Math on students' math achievement and course performance?	Extant district or state administrative data
5. To what extent does the impact of Citizen Math differ across school contexts, teacher and classroom characteristics, and student characteristics?	All data for RQs 1–4
6. To what extent is the impact of Citizen Math on student outcomes mediated by instructional activities and opportunities to learn?	
Implementation analyses	
7. To what extent are the key components of Citizen Math (e.g., the lessons, community of practice and other supports for teachers) implemented with fidelity?	Training sign-ins, Citizen Math log-in and download data, teacher lesson reflection survey
8. How does implementation of the key components of Citizen Math differ across school contexts and teacher and classroom characteristics? What factors hinder or facilitate the implementation of Citizen Math?	

The evaluation research questions align with the project's objectives and strategies and will be addressed with data collected from 120 middle schools in diverse settings (50 urban schools from Chicago Public Schools, and 70 rural from Maine and West Virginia). Half the

schools will be randomly assigned to implement Citizen Math during the project's second and third years (i.e., 2023–24 and 2024–25), and the other half will continue with business-as-usual math instruction during that period using the school's typical math curriculum. The proposed study is powered for a minimum detectable effect size of 0.13 to 0.14 for student achievement outcomes in the first and second intervention years, respectively; 0.15 to 0.17 for social-emotional learning outcomes; and 0.30 to 0.38 for teacher instructional activities. (See Appendix I.1 for details about the power analysis and Appendix I.2 for details about the statistical models for the impact analyses.)

E.1. Evaluation Methods Designed to Meet WWC Standards Without Reservations

The evaluation of the impact of Citizen Math will be based on a school-level blocked randomized controlled trial (RCT), where participating schools will be randomly assigned to the treatment or control condition within blocks defined by district (or geographic regions in rural states). To the extent that school size and average math achievement (based on the prior year's middle-grade test scores) vary substantially within district/region, we will incorporate these factors into the blocking process to create more homogeneous blocks. In schools assigned to the **treatment condition**, middle grade teachers will be expected to half or more of Citizen Math *Lessons* (at least 6 out of 12 available lessons) during each of the 2023–24 and 2024–25 school years. Teachers will participate in four hours of virtual synchronous training in the summer of 2023. The virtual training will be led by Citizen Math facilitators and the hours will likely be distributed in two-hour blocks, based on district and teacher scheduling preferences. Teachers will also have access to an online community of practice to collaborate, learn, and share best practices, as well as access to additional supports including lesson guides, video cases, and supplemental virtual synchronous and unsynchronous professional learning workshops. The

optimal support structure will be answered during the scaling work in Years 1 and 2. Because within-class instructional time is fixed, treatment teachers will substitute the Citizen Math *Lessons* for other lessons or activities that they would otherwise implement as part of the business-as-usual curriculum.

For this evaluation, schools are the appropriate unit of assignment because the Citizen Math training encourages collaboration among teachers and the *Lessons* can be easily shared among teachers within the same school. A school-level random assignment design is more likely to mitigate spillover effects compared to a teacher-level assignment design. Based on our prior experience with school-level randomized studies, we expect minimal school-level attrition over the two-year intervention.[2] The primary impact analyses for student outcomes will estimate intent-to-treat (ITT) effects among students present in the study schools at the start of the first intervention year (i.e., exclude "joiners" as defined by WWC). Within each participating school, the evaluation will focus on all grade 6–8 math teachers and their students during the first intervention year (2023–24) and will follow students over time focusing on grade 7 and 8 math classes during the second intervention year (2024–25).[3] Student rosters will be collected at the start of the 2023–24 school year to identify students in the ITT student impact sample. To get a comprehensive assessment of the instructional activities study students experience, in both

[2] An IES-funded school-level random assignment design on the impact of providing teachers and principals with performance feedback completed by AIR (Garet et al., 2017), included 127 schools in total, and only one school dropped out of the study during the two-year intervention.

[3] We will also conduct a set of supplemental impact analyses for the first and second intervention years that include all grade 6-8 students (including joiners) in the study schools at the time of outcome data collection.

implementation years the primary impact analysis for teacher instructional activities will estimate ITT effects among all grade 6–8 math teachers.[4]

Given that the proposed evaluation is based on a school-level RCT that is expected to have low cluster-level attrition and a student analytic sample where joiner bias is not a threat, the evaluation has the potential to produce strong evidence about the impact of Citizen Math. In addition, we anticipate manageable levels of student attrition over the two implementation years (i.e., about 20%) and minimal differential student attrition across conditions (i.e., less than 5 percentage points), so the student impact analyses will likely meet WWC standards without reservations. Appendix I.7 includes additional details on moderator analyses.

E.2. Guidance About Effective Strategies Suitable for Replication or Testing in Other Settings

The proposed evaluation will generate useful guidance about effective strategies for implementing and scaling Citizen Math in diverse settings by (1) including a large sample representing diverse schools in urban and rural settings; (2) deliberately assessing whether the effects of Citizen Math differ for different types of students, teachers/classrooms, and schools; (3) collecting and analyzing rich data on program implementation from multiple sources; and (4) including a cost analysis to provide valuable information about the cost-effectiveness of the program.

Diverse Settings. The commitment to this project of partner math leaders in Chicago Public Schools, Maine, and West Virginia will allow us to evaluate Citizen Math as implemented across 120 economically disadvantaged schools in both urban and rural settings.

[4] We will conduct a sensitivity analysis of teacher instructional activities that excludes teacher joiners and can be used to meet WWC standards without reservations.

Findings based on this diverse study sample will provide valuable guidance for future replications of Citizen Math in a variety of settings.

Differential Impact Analyses. The evaluation will include differential impact analyses (RQ5) to assess the extent to which the effects of Citizen Math are moderated by the characteristics of students, teachers/classrooms, and schools (see Exhibit 7). Results from these exploratory analyses will be crucial in guiding future efforts to scale Citizen Math, as they may identify settings and populations where the program is particularly effective or not well suited.

Exhibit 7. Potential Moderators at the Student, Teacher/Classroom, and School Levels

Student Level	Teacher/Classroom Level	School/District Level
Race/ethnicity, eligibility for free or reduced-priced lunch, English learner status, special education status, and prior math achievement	Years of teaching experience, years teaching middle grade math, class size, classroom average prior math achievement	State, locale (urban/rural), school size, school average prior math achievement, percentage of students of color, and percentage of students eligible for free or reduced-priced lunch

Analyses of Implementation Data from Multiple Sources. To determine how the program's key components are implemented and to provide lessons learned for future replications or testing of Citizen Math in other settings, the evaluation team will collect and analyze rich implementation data from multiple sources. Implementation data will be collected in all treatment schools during the first and second intervention years. These data include artifacts (e.g., sign-in sheets and agendas) from teacher trainings and team meetings to determine participation and coverage topics, data from Citizen Math on the number and types of lessons and implementation resources used and downloaded (i.e., lesson guides, videos of case studies, short videos and virtual workshops), and teacher log-ins and postings to the online community platform. In addition, we will have treatment teachers complete a short "lesson reflection" survey after they conduct each Citizen Math lesson. We will use the lesson reflection survey for a more detailed analysis of how teachers used the Citizen Math lessons and the

resources they accessed to support implementation. Using these data sources, we will examine the fidelity of implementation (RQ7) for both the Citizen Math lessons and the implementation supports, as well as identify factors associated with relatively poor or strong implementation of the scaling strategy and the program (RQ8).

Cost Effectiveness. To provide information about the cost of replicating Citizen Math and whether it is cost effective, the evaluation includes a cost analysis based on the Resource Cost Model (RCM; Levin & McEwan, 2002). Our analyses will identify the costs associated with implementing each component of the program, distinguish start-up costs from ongoing costs, and convert total costs to per-student costs. We will then combine the cost information and effect size estimates to describe the impact of Citizen Math on a per dollar basis.

E.3. Components, Mediators, Outcomes, and Acceptable Implementation Thresholds

The design of the proposed evaluation is informed by clearly articulated key components, mediators, and outcomes of Citizen Math as depicted in the logic model presented in Appendix G.1. As the exhibit shows, the key components of the Citizen Math program include unique content features embedded in the Citizen Math lessons and implementation supports for teachers. Together, these components are designed to improve instructional activities and students' opportunities to learn, which in turn, **mediate** the Citizen Math program's impact on both students' social-emotional outcomes and achievement outcomes (e.g., math achievement and course performance). The evaluation will include moderation analyses (RQ5) and mediation analyses (RQ6) to explore the relationships among implementation context, intermediate outcomes (i.e., teachers' instructional practices and students' opportunities to learn), and students' social-emotional learning and achievement outcomes.

Key components and measurable implementation thresholds. The evaluation will measure the following key program components with their associated proposed fidelity markers[5] and data sources (see Exhibit 8).

Exhibit 8. Key Program Components, Fidelity Markers and Data Sources

Program Component	Fidelity Marker	Data Source
Attendance to summer kickoff virtual workshop	Teacher attends kickoff	Training sign-ins
Use of lessons	Teacher uses five or more lessons	Reflection surveys, Citizen Math log-in and download data
Use of lesson guides	Teacher uses five or more lesson guides	
Use of video cases	Teacher uses one or more video cases	
Use of short videos	Teacher uses one or more short videos	
Access and collaboration in the online community of practice	Teacher access community of practice twice or more per semester	
Attendance to supplemental virtual workshops	Teacher attends one or more synchronous or asynchronous workshops	

Based on prior research on fidelity in RCTs (Durlak & DuPre, 2008; Hill & Erickson, 2019), we propose the following initial implementation fidelity thresholds for each key program component: low fidelity (less than 60% of study teachers in treatment schools participate/complete/use the program component at or above the fidelity marker), moderate fidelity (60%–80%), and high fidelity (above 80%). These thresholds will be used to assess implementation fidelity (see Strategy 2.4 in Exhibit 1).

Outcomes. The impact analyses will use multiple well-established, valid and reliable measures of (1) **instructional activities**, (2) **opportunities to learn**, (3) **student social-emotional learning** and (4) **student achievement** outcomes.

[5] Fidelity markers will be revised as needed based on lessons learned during the scaling work in Years 1 and 2 while testing strategies to enhance adoption, use, and sustainability of Citizen Math (see Objective 1 in Exhibit 1).

(1) Nature of Instructional Activities. As outlined in the logic model shown in Appendix G.1, Citizen Math is designed to improve instructional activities in math classes. To measure the quality of instructional activities (RQ1), AIR will administer a teacher log twice per semester during the first and second intervention years to provide an accurate picture of the instructional activities over the entire school year. Primary analyses will average responses across four logs for each year. Prior studies of teacher logs indicate that they can be a valid and reliable measure of instruction (Rowan & Correnti, 2009). The log will include the following measures adapted from a RAND study of inquiry-based instruction (Le, Stecher, Lockwood, Hamilton, & Robyn, 2006): *inquiry-based practices* intended to actively engage students and promote problem solving skills ($\alpha = 0.83$), *inquiry-based activities* intended to facilitate critical thinking ($\alpha = 0.77$), *discussion* ($\alpha = 0.74$), and *mathematical processes* that include multiple representations and develop conceptual understanding ($\alpha = 0.58$). Together, these measures capture the types of instructional activities Citizen Math intends to promote in the classroom. See Appendix I.2 for a detailed description of each measure.

(2) Student Opportunities to Learn. To measure students' opportunities to learn in their math class (RQ2), AIR will administer a student survey at the end of each intervention year (2023–24 and 2024–25). The survey will include five opportunities-to-learn measures adapted from prior AIR studies that provided evidence of sufficient validity and reliability for the measures (Rickles, et al., 2019; Walters et al., in press): *opportunities to make real-world connections* (reliability = 0.84), *opportunities to justify mathematical reasoning* ($\alpha = 0.82$), *opportunities to solve challenging math problems* ($\alpha = 0.78$), and *opportunities to demonstrate conceptual understanding* ($\alpha = 0.80$). (See Exhibit I.3 in Appendix I for a detailed description of each measure.)

(3) Student Social-Emotional Learning. To measure student social-emotional learning that will promote thoughtful and productive citizenship (RQ3), AIR will include items on the student survey for five valid and reliable measures adapted from prior studies (Gehlbach & Hough, 2018; Heppen et al., 2017; Rickles et al., 2019): *motivation to learn* (reliability = 0.75); *behavioral engagement* ($\alpha = 0.72$); *confidence in mathematics* (reliability = 0.90) as a component of self-esteem; *social awareness* ($\alpha = 0.82$) as a component of positive personal relationships with others; and *creative thinking* (reliability = 0.77) as a component of problem solving skills. (See Exhibit I.4 in Appendix I for a detailed description of each measure.)

(4) Student Achievement Outcomes: Math Achievement and Course Performance. To measure students' math content knowledge and performance within the context of grade 6–8 math courses (RQ4), AIR will rely on each grade's end-of-year state math assessment for each intervention year (2023-24 and 2024-25) and students' end-of-semester course grades. Scaled scores in state assessments will be standardized within grade and state to make them comparable. As standardized tests, the state assessments are considered valid and reliable by the WWC standards. Course grades will be used as a supplemental measure of student achievement. Because grading practices differ from teacher to teacher, course grade will not be interpreted as a precise measure of student learning. Rather, it will be interpreted as a marker of course performance, which is policy relevant and a strong indicator of future academic success (Allensworth & Clark, 2020).

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